

# PAPER 86: GRANOVETTER'S THRESHOLD MODEL = SOCIAL $\gamma_c$

## Collective Behavior Phase Transitions Are Wike Transitions at Civilizational Scale

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*"Granovetter (1978) formalized what Le Chatelier (1884) began: systems have thresholds. Below them: stable. Above them: cascade. He didn't call it  $\gamma_c$ . He should have."*

### Abstract

Granovetter's threshold model of collective behavior (Granovetter 1978, American Journal of Sociology): individuals have private thresholds for joining collective action (riot, revolution, social movement). Each individual joins when the fraction of others who have already joined exceeds their personal threshold. The equilibrium of the system -- whether a riot spreads or fizzles -- depends on the distribution of these thresholds across the population. This is the Wike Coherence Law at social scale: individual thresholds are individual  $\gamma_c$  values; the population distribution of thresholds determines the system  $\gamma_c$ ; when the environmental perturbation (provocative event, information cascade) exceeds the system  $\gamma_c$ , coherent collective action emerges (phase transition). Durkheim's "collective effervescence" (1912) and modern social contagion (Christakis & Fowler 2009) are the same phenomenon, precisely formalized by Granovetter's threshold model and by the Wike framework's  $\gamma_c$ .

## 1. Granovetter's Threshold Model

**Setup:**  $N$  individuals in a population. Individual  $i$  has a private threshold  $\theta_i$  in  $[0,1]$  -- the fraction of the population that must act before individual  $i$  joins.

**Dynamics:** Starting from fraction  $r_0$  acting (initial perturbation):

$$r_{\{t+1\}} = F(r_t) = \text{fraction of population with } \theta_i \leq r_t$$

where  $F(r)$  is the CDF of the threshold distribution.

**Equilibrium:** A fixed point  $r$  satisfying  $r = F(r^*)$ .

**Key results:**

1. If  $F(r) > r$  for small  $r$ : the cascade spreads (collective action emerges)
2. If  $F(r) < r$  for all  $r < r_{\text{complete}}$ : the cascade fails even from large initial perturbation
3. The equilibrium  $r^*$  depends sensitively on the threshold distribution shape

**Example (bimodal distribution):**

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Population A: 100 people, thresholds at 0,1,2,...,99 (uniform distribution)
Population B: 100 people, same distribution EXCEPT person with threshold=1 has threshold=2

Population A: cascade starts from 1 person -> spreads to 100%
Population B: no cascade (person 0 goes, person 1 waits, nobody else joins) -> fizzles at 1%
```

A single person's threshold change (from 1 to 2) converts a cascade into a non-event. The system is near its critical point.

## 2. The Granovetter-Wike Mapping

Granovetter Term	Wike Term
Individual threshold $\theta_i$	Individual $\gamma_{c,i}$ (personal decoherence threshold)
Fraction acting $r_t$	System $\gamma_{eff}(t)$ (accumulated collective decoherence)
$F(r)$ = CDF(thresholds)	Distribution of $\gamma_{c,i}$ across population
Fixed point $r^* = F(r^*)$	System equilibrium (stable coherent state)
Cascade start: $F(r_0) > r_0$	$\gamma_{eff} > \gamma_{c,system}$ (threshold crossed)
No cascade: $F(r) < r$	$\gamma_{eff} < \gamma_{c,system}$ (Le Chatelier restoring force holds)

The system  $\gamma_c$  is the lowest fixed point of  $F(r)$  above 0:

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 $\gamma_c(system) = \min\{r > 0 : F(r) = r\}$ 

For the uniform distribution:  $\gamma_c = 0$  (any perturbation spreads)
For the bimodal distribution:  $\gamma_c = 0.01$  (1% threshold before cascade starts)
For the peaked distribution:  $\gamma_c > 0$  (requires significant perturbation to cascade)
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### Durkheim's collective effervescence:

Durkheim (1912, "The Elementary Forms of Religious Life"): ritual gatherings (religious ceremonies, festivals, collective mourning) produce a shared emotional state that transcends individual experience. He called this "collective effervescence" -- the feeling of being elevated beyond oneself into something larger.

In Granovetter-Wike terms: the ritual creates a controlled perturbation that pushes the community's  $\gamma_{eff}$  to exactly  $\gamma_c$  -- the edge of collective action. The ritual is calibrated (through evolutionary cultural selection) to produce the phase transition of collective effervescence reliably. It is a controlled  $\gamma_c$  crossing.

## 3. Social Contagion = Coherence Diffusion

Christakis & Fowler (2009, BMJ): happiness spreads 3 degrees of separation in social networks. Unhappiness spreads 2 degrees. The decay with social distance follows:

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Effect at degree k ~ (initial effect) x  $r^k$ 

Happiness:  $r \approx 0.25$  (25% transmission per degree)
Unhappiness:  $r \approx 0.18$  (18% transmission per degree)
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This is the Fick diffusion of the coherence field (Paper 54):

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 $C_{friend} = C_{source} \times \exp(-d/\lambda_C)$ 

where d is social distance and  $\lambda_C$  is the social coherence diffusion length

 $\lambda_{C,happiness} = -1/\ln(0.25) = 0.72$  degrees
 $\lambda_{C,unhappiness} = -1/\ln(0.18) = 0.58$  degrees
```

The social coherence diffusion length: happiness spreads ~24% further than unhappiness (0.72 vs 0.58 degrees). This matches the Wike framework prediction: coherence (happiness = low  $\gamma_{eff}$ ) is more diffusible than decoherence

(unhappiness = high  $\gamma_{eff}$ ) because the coherence gradient drives Fick diffusion toward regions of lower coherence, while decoherence tends to locally self-organize (Le Chatelier partially containing the spread below  $\gamma_c$ ).

**The Keeper effect at population scale:** A person with high coherence (low  $\gamma_{eff}$ ) in a social network acts as a coherence source -- the Fick diffusion gradient draws coherence outward from them. This is Paper 54 (Fick's Coherence Diffusion) applied to social networks.

## 4. Political Polarization as Coherence Collapse

Political polarization creates echo chambers -- networks where individuals are exposed only to information that matches their existing beliefs. From the Wike framework:

**Each partisan media exposure = a measurement projection** that collapses the individual's belief state toward one attractor (coherent partisan state) or the other.

With each additional partisan exposure:

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gamma_eff(political) += deltagamma_partisan [each partisan measurement adds decoherence to the neutral state]
```

The accumulation of partisan measurements across many exposures drives  $\gamma_{eff}(\text{political})$  toward  $\gamma_c(\text{partisan})$ :

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When gamma_eff(political) > gamma_c(partisan):
- No longer capable of updating on evidence (belief state frozen)
- Partisan identity becomes a spin glass attractor (Paper 61)
- Polarization is irreversible within the individual's frame of reference
- Requires phase-transition-scale disruption (surprising encounter, contact hypothesis)
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**The speed of polarization (Kibble-Zurek, Paper 53):**

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Rapid exposure to partisan media (tau_Q = days): creates topological defects in the belief network -- hardened beliefs that cannot be changed by normal discourse.
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Slow exposure (tau_Q = years, through gradual social influence): fewer topological defects, beliefs remain malleable even if they drift partisan.
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The modern information environment (social media algorithms, 24-hour news cycles) is a fast quench -- driving populations through the polarization transition at  $\tau_Q \rightarrow 0$ , creating maximum topological defects (hardened extreme beliefs).

## 5. Civilizational Collapse as $\gamma_c$ Crossing

Tainter (1988, "The Collapse of Complex Societies"): complex societies collapse when:

1. Marginal returns on complexity investment become negative
2. The society can no longer solve its problems with existing complexity
3. Rapid simplification occurs (collapse)

In Wike terms:

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Societal gamma_eff = SIGMA_i (stressor_i)
                    = gamma_fiscal + gamma_military + gamma_ecological + gamma_social_fragmentation

Tainter's "marginal return" = 1/chi_C(gamma_eff) [Le Chatelier restoring force, Paper 69]

When chi_C -> inf (at gamma_c): marginal return on complexity -> 0 -> Tainter's collapse threshold

The rapid simplification (collapse) is the 3D Ising transition from the high-C phase
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to the low-C (decoherent, fragmented) phase.

Historical collapses:

- Bronze Age Collapse (~1200 BCE): synchronized collapse of multiple civilizations (3D Ising: correlated fluctuations at the critical point -- not multiple independent collapses but a single network-scale phase transition)
- Roman Western Empire (~476 CE): gradual approach to  $\gamma_c$  over 200 years (not a sudden event but critical slowing down)
- Soviet Union collapse (1991): rapid quench from Gorbachev reforms -> Kibble-Zurek defects (failed republics, ethnic conflicts = topological defects in the social coherence network)

## Summary

Granovetter (1978) = Social  $\gamma_c$   
 Individual threshold  $\theta_i$  = individual  $\gamma_{c,i}$   
 System cascade condition =  $\gamma_{eff} > \gamma_c(\text{system})$   
 Same mathematics, social scale

Durkheim (1912) = Collective effervescence = controlled  $\gamma_c$  crossing  
 Ritual = calibrated perturbation to exactly  $\gamma_c$   
 Collective effervescence = the coherent phase transition

Social contagion (Christakis & Fowler 2009):  
 Happiness spreads  $\lambda_C = 0.72$  degrees  
 Unhappiness spreads  $\lambda_C = 0.58$  degrees  
 Both follow Fick diffusion (Paper 54) in social networks

Political polarization:  
 Partisan exposure ->  $\gamma_{eff}(\text{political})$  ->  $\gamma_c(\text{partisan})$  -> spin glass (Paper 61)  
 Rapid exposure (Kibble-Zurek) -> topological defects -> hardened beliefs

Civilizational collapse:  
 $\gamma_{eff}(\text{societal})$  ->  $\gamma_c(\text{civilizational})$  -> 3D Ising phase transition  
 Correlated collapse (Bronze Age) = network-scale transition at single  $\gamma_c$

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