PAPER 91: VON NEUMANN ENTROPY, SHANNON ENTROPY, AND THE COHERENCE CLIFF

gamma_c Is an Entropy Phase Transition -- Below It, Entropy Is Actively Suppressed

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> *"Shannon counted bits. von Neumann counted quantum states. The Wike Coherence Law is about both: below gamma_c, the body is paying the metabolic cost to suppress entropy. Above gamma_c, entropy wins."*

Abstract

The von Neumann entropy S = -Tr(rho log rho) is the quantum measure of coherence: S = 0 for a pure state (perfect coherence), S = log d for the maximally mixed state (complete decoherence). The Wike Coherence Law C = C_0 x exp(-alphagamma_eff) maps directly to von Neumann entropy:

```
S(gamma_eff, t) = log(2) - C_0 x exp(-2alphagamma_eff x t) x log[(1+C(t))/(1-C(t))] / 2 + ...

At gamma_eff = 0: S -> 0  (pure state, perfect coherence)
At gamma_eff -> inf: S -> log(2)  (maximally mixed, complete decoherence)
At gamma_eff = gamma_c: S_c = crossover entropy
```

Shannon entropy H (classical information theory) is the classical limit of von Neumann entropy. For REQMT measurements (Paper 05): lower Shannon entropy across all 5 channels simultaneously is the measurable signature of low gamma_eff. The C_alive distribution (Paper 59) has a maximum entropy at T* = body temperature that provides the optimal balance between coherent information (low S) and accessible thermal energy (high T). The framework predicts: healthy subjects show lower multi-modal signal entropy than stressed subjects, and this entropy difference is the macroscopic signature of the gamma_eff difference between the two states.

1. Von Neumann Entropy

For a quantum state rho:

```
S(rho) = -Tr(rho log rho)

Pure state (|psi><psi|): S = 0
Mixed state (1/d x I): S = log d
```

For a single qubit (d=2):

```
rho = [[1/2 + C/2, 0], [0, 1/2 - C/2]]  [dephasing channel, Paper 68]

S(C) = -((1+C)/2) log((1+C)/2) - ((1-C)/2) log((1-C)/2)
```

Taylor expansion near C = 0:

```
S(C) ~= log(2) - C^2/2 - C^4/12 - ...

At small C (near decoherent phase):
  S ~= log(2) [maximum entropy]
```

Taylor expansion near C = 1 (near pure state):

```
S(C) ~= (1-C)/2 x log(2/(1-C)) -> 0 as C -> 1
```

The Wike Law in entropy language:

```
C(t) = C_0 x exp(-alphagamma_eff x t)

S(t) = -((1+C(t))/2) x log((1+C(t))/2) - ((1-C(t))/2) x log((1-C(t))/2)

dS/dt = alpha x gamma_eff x C(t) x log((1+C(t))/(1-C(t))) > 0  for all t > 0

Entropy INCREASES monotonically over time at fixed gamma_eff.
```

2. gamma_c as Entropy Regulation Threshold

Below gamma_c (coherent phase):

```
The system actively maintains C > 0 through the Bootstrap Loop (Paper 02).
The Bootstrap Loop is a Maxwell's Demon process (Paper 70): it pays the Landauer cost
to suppress entropy S below the equilibrium value.

Entropy production rate from environment: dS_env/dt = alphagamma_eff x C x (log term)
Bootstrap entropy suppression rate: dS_Bootstrap/dt = -k_ATP x PATP

Coherent steady state: dS_env/dt = dS_Bootstrap/dt

At gamma_eff = gamma_baseline = 0.001:
  S is maintained at S_baseline << S_max (the system pays metabolic cost to suppress entropy)
```

At gamma_eff = gamma_c:

```
The Bootstrap Loop cannot suppress entropy fast enough:
  dS_env/dt > dS_Bootstrap/dt (maximum)

S begins increasing irreversibly -> C drops irreversibly -> wind-up (Paper 16)
The system crosses the entropy "maintenance threshold."
```

Above gamma_c:

```
S -> S_max = log(2) (maximum entropy, complete decoherence)
Bootstrap Loop cannot restart (Paper 63: C_0 -> 0 for phi < phi_c)
Entropy production wins: S -> S_max regardless of metabolic input
```

Summary: gamma_c is the entropy regulation threshold -- below it, the body CAN suppress entropy (costs ATP), above it, the body CANNOT suppress entropy regardless of energy input.

3. Shannon Entropy and REQMT

Shannon entropy for a discrete distribution {p_i}:

```
H = -SIGMA_i p_i x log(p_i)
```

```
For a time series signal x(t):
  H_temporal = -SIGMA p(x) log p(x)  [entropy of value distribution]
  H_spectral = -SIGMA S(f)/S_total x log(S(f)/S_total)  [spectral entropy]
```

Prediction for REQMT (Paper 05):

For a subject at gamma_eff = gamma_baseline (healthy, coherent):

```
HRV time series: quasi-periodic (fractal 1/f, alpha ~= 1.0-1.2) -> LOW spectral entropy
Thermal IR: spatially uniform with minor oscillations -> LOW spatial entropy
Vocal signal: harmonic structure (fundamental + overtones) -> LOW spectral entropy
rPPG: coherent photoplethysmography -> LOW temporal entropy
Skin conductance: slow drift with coherent events -> LOW spectral entropy

All 5 channels: LOW entropy simultaneously (coherent state)
```

For a subject at gamma_eff -> gamma_c (stressed, approaching collapse):

```
HRV: irregular (Gaussian, alpha ~= 0.8) -> HIGH spectral entropy
Thermal IR: spatially non-uniform (stress thermography) -> HIGH spatial entropy
Vocal: decreased harmonic structure (stress vocalization) -> HIGH spectral entropy
rPPG: noisy, irregular -> HIGH temporal entropy
Skin conductance: random spiky -> HIGH spectral entropy

All 5 channels: HIGH entropy simultaneously (decoherent state)
```

The REQMT entropy prediction (testable, E5 in updated UNANSWERED_QUESTIONS):

```
E_REQMT = SIGMA_k H_k(signal_channel_k) / k_total   [average across 5 channels]

Healthy subjects (gamma_eff ~= gamma_baseline):  E_REQMT < E_threshold
Stressed subjects (gamma_eff -> gamma_c):        E_REQMT > E_threshold

The entropy difference: DELTAE = E_REQMT(stressed) - E_REQMT(healthy) > 0

DELTAE ~ (gamma_eff - gamma_baseline) x alpha x t_REQMT

DELTAE/E_REQMT(healthy) = (gamma_stressed - gamma_baseline) / gamma_baseline = 10 (for fight/flight
vs HeartMath)
Expected: stressed subjects show ~10x higher multi-modal entropy than HeartMath subjects.
Observed in simulation: C_stressed/C_calm = 2.3x (not 10x, due to distribution averaging, Paper 81).
Expected DELTAE: ~3-5x (between the theoretical 10x and the noise-averaged 2.3x).
```

4. The Bekenstein Bound -- Maximum Entropy in the Coherent Volume

The Bekenstein bound (Bekenstein 1973): the maximum entropy that can be stored in a physical system of energy E and radius R is:

```
S_max <= 2piER / (h-barc)   [Bekenstein bound]
```

For a human brain (E ~= 20 W x tau_coherence, R ~= 0.1 m):

```
At T_coherence ~= 1 ms (neural timescale):
  E_coherence = 20 W x 0.001 s = 0.02 J
  S_max = 2pi x 0.02 x 0.1 / (1.055x10^-^3^4 x 3x10^8) ~= 4x10^2^7 bits
```

The Bekenstein bound is astronomically larger than the ~10^1^4 synaptic bits in a brain -- the fundamental limit is nowhere near the biological limit. The effective maximum is set by the **Landauer limit** (Paper 70) and the metabolic cost of entropy suppression, not by the Bekenstein bound.

At gamma_c: The entropy per coherence domain approaches k_B x ln(2) (one bit per degree of freedom). At gamma_c crossing, the system loses one bit of coherence per quantum degree of freedom -- the topological transition corresponds to an entropy increase of DELTA N_DOF x k_B x ln(2).

5. Emotional Gate Operators in Entropy Language

Paper 07 maps emotions to quantum gate operations:

```
Love/joy/peace -> Unitary gates (U+U = I) -> S unchanged (entropy conserved)
Fear/collapse  -> Non-unitary measurement -> S increases (entropy produced)
```

In information theory:

```
Unitary gate = reversible operation = no entropy production = Shannon channel capacity preserved
Non-unitary measurement = irreversible = entropy production = Shannon capacity reduced

Love: H(output) = H(input)  [entropy preserved]
Fear: H(output) > H(input)  [entropy added from environmental noise]
```

The entropy cost of fear:

Each fear-response event (a gamma_eff spike, Paper 55, Anti-Zeno):

```
DELTAS_fear = k_B x (gamma_fear - gamma_baseline) x t_exposure x (dS/dgamma)|_{gamma_eff}
```

Accumulated over a lifetime of chronic stress:

```
S_accumulated = integral DELTAS_fear x n_events x t_events dt
             ~= k_B x (gamma_stress - gamma_baseline) x T_lifetime x alpha x C_0
```

Physical entropy cost of chronic stress:

```
T_lifetime = 70 years = 2.2x10^9 s
gamma_stress - gamma_baseline = 0.001 (chronic elevated by 2x)
alpha = 1000, C_0 = 0.85

S_accumulated ~= k_B x 0.001 x 2.2x10^9 x 1000 x 0.85 ~= 1.87x10^9 k_B

At T = 310K:
Q_accumulated = T x S_accumulated ~= 310 x 1.38x10^-^2^3 x 1.87x10^9 ~= 8x10^-^1^2 J

This is tiny in macroscopic terms -- but in quantum terms, it represents:
  1.87x10^9 k_B / k_B = 1.87x10^9 nat ~= 2.7x10^9 bits of information entropy
  accumulated over a lifetime of chronic stress
```

This is the "weight" of chronic stress in information-theoretic units. It is a concrete, calculable number.

Summary

```
Von Neumann entropy and Wike Coherence Law:
  S(C) = -(1+C)/2 x log(1+C)/2 - (1-C)/2 x log(1-C)/2
  S = 0 at C = 1 (perfect coherence)
  S = log(2) at C = 0 (complete decoherence)
  dS/dt > 0 always (entropy increases, coherence decreases, unless Bootstrap active)

gamma_c as entropy regulation threshold:
  Below gamma_c: Bootstrap suppresses entropy (pays Landauer price in ATP)
  At gamma_c: Bootstrap maximum entropy suppression rate = environmental entropy production
  Above gamma_c: entropy cannot be suppressed regardless of energy input

REQMT entropy prediction (E5):
  Healthy: all 5 channels LOW entropy simultaneously
```

```
  Stressed: all 5 channels HIGH entropy simultaneously
  Expected DELTAE: ~3-5x between stressed and HeartMath conditions

Shannon entropy of REQMT:
  E_REQMT = multi-channel average spectral/temporal entropy
  Direct measurement of gamma_eff via entropy differential
```

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